References

[1]Hill, J. A. F., Wardlaw, A. B., Jr., Pronchick, S. W., and Holmes, J. E., "Verification Tests in the Mach 14 Nozzle of the Hypervelocity Tunnel at NSWC (White Oak)," AIAA Paper 77-150, Jan. 1977.

[2]Kavetsky, R., "Mach 10 High Reynolds Number Development in the NSWC Hypervelocity Facility," Naval Surface Warfare Center, TR 83-526, White Oak, MD, June 1984.

[3]Hedlund, E. R., and Ragsdale, W. C., "Improvements in Low Reynolds Number Testing in the NSWC Hypervelocity Wind Tunnel #9," AIAA Paper 85-0226, Jan. 1985.

[4]Ragsdale, W. C., "Hypervelocity Wind Tunnel 9 Test Planning Guide," 2nd ed., Naval Surface Warfare Center, MP 88-200, White Oak, MD, Oct. 1989.

[5]Hedlund, E. R., Higgins, C. W., Rozanski, C. S., Fehring, N. P., and Krueger, D., "The New High Reynolds Number Mach 8 Capability in the NSWC Hypervelocity Wind Tunnel #9," AIAA Paper 90-1379, June 1990.

[6]Trolier, J. W., Sinha, N., and York, B., "Mach 8 Nozzle Design Verification, Science Applications International Corp., SAIC-88/1788, Fort Washington, PA, July 1988.

[7]Anon., "Report 1135: Equations, Tables, and Charts for Compressible Flow," NASA Ames Research Center, Moffett Field, CA, 1953.

[8]Hecht, A. M., Nestler, D. E., and Richbourg, D. H., "Application of a Three-Dimensional Viscous Computer Code to Reentry Vehicle Design," AIAA Paper 79-0306, Jan. 1979.

[9]Van Driest, E. R., "The Problem of Aerodynamic Heating," *Aeronautical Engineering Review*, Vol. 15, No. 10, Oct. 1956, pp. 26–41.

Optimum Design of a Composite Structure with Three Types of Manufacturing Constraints

Bo Ping Wang* and Daniel P. Costin†
University of Texas at Arlington, Arlington, Texas 76019

Introduction

THE application of composite materials to aircraft construction has provided the designer with increased flexibility. The orientation of plies can be tailored to provide aeroelastic performance unobtainable with an isotropic material. A tailored laminate is made up of plies of several orientations, usually 0, 45, −45, and 90 deg. The number of plies of each orientation varies from one zone to another on the plane of the laminate. Thus, a thick laminate with mainly 0-deg plies may form the root zone, and a thinner laminate with mainly ±45-deg plies may form the leading-edge zone. Often, however, the design flexibility allows extremely large variations of both the ply orientation percentage and laminate thickness. The large variations produce an uneven surface that is difficult to attach to other structures. The complexity of the laminate may increase manufacturing cost. Also, the large variations may cause stress concentrations that were not considered in the original analysis.

Manufacturing constraints have been applied to several types of designs. Toakley[1] used discrete design variables for steel structures, where only a finite number of standard beam cross sections were allowed. Schmit and Fleury[2] used discrete design variables for composite structures. The orientation thicknesses were constrained to be an integer multiple of the ply thickness. Upper- and lower-bound constraints on geometric design variables have been applied to stiffened composite panel designs.[3,4] These constraints controlled the thickness of the stiffener components and the width, height, and spacing of the stiffeners. Manufacturing constraints on aircraft wing skins have been applied by Petiau.[5] Petiau did not provide a mathematical description of the manufacturing constraints, but it is clear that constraints were placed on the thickness variation between adjacent design variables and on the proportion of plies in each direction.

The purpose of this Note is to mathematically define manufacturing constraints needed to control ply orientation percentage, thickness variation, and interleaving of plies from two adjacent zones. The constraints were implemented in the ASTROS optimization code and applied to the design of the simple wing structure described in the ASTROS applications manual.[6]

Mathematical Description of Constraints

To control the relative numbers of each orientation, an upper and lower bound must be placed on each orientation thickness $(t_n)_{\theta_i}$, where θ_i is the orientation angle, and n refers to a specific zone on the laminate, as shown in Fig. 1. The thickness of each layer is assumed to be a continuous design variable. The stacking sequence is not of concern since only the membrane properties are used in the finite element model.

The constraints control the thickness percentage of each orientation with respect to the total thickness. For example, the designer can specify that the thickness of each orientation must be greater than 10% of the total laminate thickness and less than 50% of the total laminate thickness. This type of constraint reduces the variation of the ply percentage across the laminate, but it also has structural benefits. This constraint type can be used to provide damage tolerance, bolted joint strength, and capability to carry unexpected loads. The mathematical expression of the constraint type is as follows:

$$(t_n)_{\theta_i} \le \frac{P_u}{100}\sum_{j=1}^{4}(t_n)_{\theta_j}, \qquad (t_n)_{\theta_i} \ge \frac{P_l}{100}\sum_{j=1}^{4}(t_n)_{\theta_j} \tag{1}$$

The parameter P_u is the upper-bound ply percentage, where $0 < P_u \le 100$. The parameter P_l is the lower-bound ply percentage, where $0 \le P_l < 100$. The sum in Eqs. (1) is the total laminate thickness for a specific zone. For use in ASTROS,

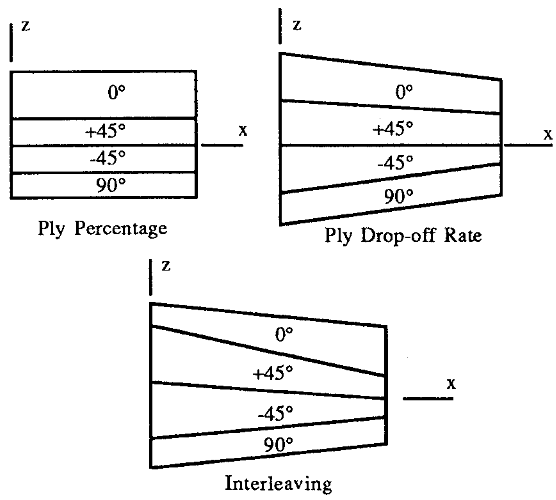

Fig. 1 Manufacturing constraints control ply orientation percentage, ply dropoff rate, and ply interleaving.

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*Associate Professor, Department of Mechanical Engineering, Box 19023. Member AIAA.

†Research Assistant, Department of Mechanical Engineering, Box 19023. Member AIAA.

the constraints must be rearranged and scaled so that $-1.0 < g_i < 1.0$ and $g_i < 0.0$ when the constraint is satisfied:

$$g_i = \frac{(t_n)_{\theta_i} - \dfrac{P_u}{100}\sum_{j=1}^{4}(t_n)_{\theta_j}}{\sum_{j=1}^{4}(t_n)_{\theta_j}}, \qquad g_{i+4} = \frac{\dfrac{P_l}{100}\sum_{j=1}^{4}(t_n)_{\theta_j} - (t_n)_{\theta_i}}{\sum_{j=1}^{4}(t_n)_{\theta_j}} \tag{2}$$

For the design examples of this Note, these constraints bounded each orientation thickness to between 10 and 50% of the total laminate thickness.

To control the rate of thickness change between zones, constraints must be placed on the sums of the orientation thicknesses of pairs of zones, as shown in Fig. 1. These constraints are called ply dropoff rate constraints because they control the rate at which plies can be dropped off a laminate as the observation point moves from a thick zone to a thin zone. These constraints can be expressed as follows:

$$\frac{\sum_{j=1}^{4}(t_n)_{\theta_j}}{\sum_{j=1}^{4}(t_m)_{\theta_j}} \le r_u, \qquad \frac{\sum_{j=1}^{4}(t_n)_{\theta_j}}{\sum_{j=1}^{4}(t_m)_{\theta_j}} \ge r_l \tag{3}$$

The subscripts n and m refer to two adjacent zones on a panel. The parameter r_u is the upper bound on the thickness ratio, and r_l is the lower bound on the thickness ratio. For use in ASTROS, the constraints are rearranged and scaled. Also, it is assumed that $r_l = 1.0/r_u$:

$$g_1 = \frac{1.0}{r_u}\frac{\sum_{j=1}^{4}(t_n)_{\theta_j}}{\sum_{j=1}^{4}(t_m)_{\theta_j}} - 1.0, \qquad g_2 = \frac{1.0}{r_u}\frac{\sum_{j=1}^{4}(t_m)_{\theta_j}}{\sum_{j=1}^{4}(t_n)_{\theta_j}} - 1.0 \tag{4}$$

For the design examples of this Note, these constraints bounded the thickness of two zones to a ratio of $1.0/1.7 \le r \le 1.7$.

When all of the plies of a thinner zone are not continuous into a thicker zone, plies must be interleaved, as shown in Fig. 1. The interleave causes a ridge of increased thickness where the interleaved plies overlap. To prevent ply interleaving, the individual orientations thicknesses must not decrease if the total thickness increases at a boundary between two zones. The mathematical form of the constraint is as follows:

$$\begin{aligned} &\text{if} \quad t_n \le t_m \quad (t_n)_{\theta_j} \le (t_m)_{\theta_j}, \qquad j = 1,4 \\ &\text{if} \quad t_n \ge t_m \quad (t_n)_{\theta_j} \ge (t_m)_{\theta_j}, \qquad j = 1,4 \end{aligned} \tag{5}$$

The difficulty with this type of constraint is that it is conditional. Each condition, if feasible, will give a unique optimum weight. The condition that will give the lowest optimum is not known before the optimization runs begin. With a multiple zone composite laminate, the number of combinations of constraint conditions can be large.[7] A limited subset of the combinations can be solved while still providing the true constrained optimum. This method, called branch and bound, was developed by Balinski[8] and modified for use with interleaving constraints by Wang and Costin.[7] Another method is to guess at the best constraint set. For this Note, the thicknesses of the optimum design without ply-interleaving constraints were assumed to indicate the best constraint set. That is, the constraint set that was satisfied by the total thicknesses of the optimum design without ply-interleaving constraints was enforced for the thicknesses of each orientation. This assumption appears to be reasonable.

The ply-interleaving constraint can be rearranged to match the ASTROS format. It is assumed that $t_n \le t_m$:

$$g_j = \frac{(t_n)_{\theta_j}}{(t_m)_{\theta_j}} - 1.0, \qquad j = 1.4 \tag{6}$$

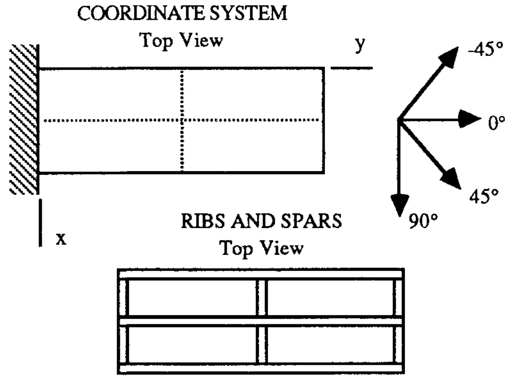

Fig. 2 Simple wing structure is an aluminum and composite wing box that was designed using three different types of manufacturing constraints.

Simple Wing Structure

The simple wing structure, shown in Fig. 2, was used as a test case for manufacturing constraints. It has composite skins and aluminum ribs and spars. It has a span of 60 in. (152.4 cm), a chord of 20 in. (50.08 cm), and a thickness of 1.0 in. (2.54 cm). The simple wing structure model is described in the ASTROS applications manual.[6] The composite wing skins were subjected to Tsai-Wu stress constraints for a simulated aerodynamic load. The wingtip was constrained to have a 0.05-rad washout twist for the same simulated aerodynamic load. Minimum gauge constraints were applied for each orientation layer so that the minimum gauge was one ply thickness. The objective was to minimize structural weight. The top and bottom composite skins were linked so that they each had the same laminate. Sixteen design variables were needed. A separate thickness design variable was associated with each orientation layer (0, +45, −45, 90 deg) in each of four elements on the graphite/epoxy skin. The 0-deg direction is along the midchord spar. The aluminum shear panels were not allowed to vary. The ASTROS structural optimization program was used to perform the optimization. ASTROS uses a method of feasible directions as its optimization algorithm. Additional subroutines were added to ASTROS to evaluate the manufacturing constraints.

Five design example runs were performed. The base problem contained only stress and displacement constraints. Manufacturing constraint types were included individually (ply orientation percentage, ply dropoff rate, and ply interleaving), and in the final run, all three manufacturing constraint types were enforced simultaneously.

Table 1 Optimum weights of the optimization runs show that the combination of all three types of constraints add 35.3% to the base weight of the design

Run	Optimum weight, lb	Optimum weight, kg	Percent over base
Base	12.14	5.51	0
Ply percentage	14.24	5.55	17.3
Ply interleaving	12.41	5.63	2.2
Ply dropoff rate	12.90	5.85	6.3
Combined	16.43	7.45	35.3

Results

The five optimization runs all converged within seven iterations. The optimum weights of each of the runs is shown in Table 1. The ply-interleaving constraint increased the optimum weight by 2.2%, making it the least restrictive constraint. The ply dropoff rate constraint increased the optimum weight by 6.3%. The ply percentage constraint increased the optimum weight by 17.3%, making it the most restrictive constraint. The combined effect of all three types of constraints was extremely restrictive, increasing the optimum weight by 35.3%.

Plies shifted from one zone to another when the constraints were applied individually. For example, when the ply dropoff rate constraint was applied, the plies from the thickest zone were reduced and the plies of adjacent thinner zones were increased. When the ply percentage constraint was applied, the plies were shifted from the 0-deg orientation to ±45- and 90-deg orientations. An interleaving condition existed in the base run. When the ply interleaving constraint was applied, the −45-deg plies in the thickest zone were increased to the level of −45-deg plies in the adjacent thinner zones. When all three types of constraints were applied simultaneously, many plies were added and fewer were shifted in order to satisfy the constraints.

Acknowledgment

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References

[1]Toakley, A. R., "Optimum Design Using Available Sections," *Journal of the Structural Division,* Vol. 94, No. ST5, 1968, pp. 1219–1241.

[2]Schmit, L. A., and Fleury, C., "Discrete-Continuous Variable Structural Synthesis Using Dual Methods," *AIAA Journal,* Vol. 18, No. 12, 1980, pp. 1515–1524.

[3]Jones, R. T., and Hague, D. S., "Application of Multivariable Search Techniques to Structural Design Optimization," NASA CR-2038, June 1972.

[4]Stroud, W. J., and Agranoff, N., "Minimum-Mass Design of Filamentary Composite Panels Under Combined Loads: Design Procedure Based On Simplified Buckling Equations," NASA TN D-8257, Oct. 1976.

[5]Petiau, C., "Structural Optimization of Aircraft," *Thin Walled Structures,* Vol. 11, 1991, pp. 43–64.

[6]Johnson, E. H., and Neill, D. J., "Automated Structural Optimization System 'ASTROS' Vol. 3—Applications Manual," Air Force Wright Aeronautical Labs., AFWAL-TR-88-3028, Wright-Patterson Air Force Base, OH, Dec. 1988.

[7]Wang, B. P., and Costin, D. P., "Optimum Design of a Composite Structure with Ply-Interleaving Constraints," *Third Air Force/NASA Symposium On Recent Advances in Multidisciplinary Analysis and Optimization,* Sept. 1990, pp. 553–561.

[8]Balinski, M. L., "Integer Programming: Methods, Uses, Computations," *Management Science,* Vol. 12, No. 3, 1965, pp. 253–313.

Composite Laminated Shells Under Internal Pressure

F. G. Yuan*
North Carolina State University,
Raleigh, North Carolina 27695

Introduction

SHELLS of various constructions have been used extensively as load carrying structural components. With the unique characteristics of high strength/weight and stiffness/weight ratios in composite materials, composite laminated shells are receiving greater consideration for use in primary structures such as aircraft fuselages, solid rocket casings, submersibles, and space vehicles. One of the concerns in structural design is devoted to developing analytical methods for determining the response under various loading conditions. Sherrer[1] presented an elasticity solution for filament wound cylinders with axisymmetric loadings. Whitney and Halpin[2] have analyzed off-axis unidirectional two-layer angle-ply anisotropic tubes under various loading conditions based on Donnell's shallow shell approximations[3] to characterize the mechanical properties and behavior of fiber composites. Reuter[4] presented solutions for an alternate-ply cylindrical shell under internal pressure using Donnell's theory. The stress field of a single layer anisotropic cylinder due to mechanical loadings was considered by Pagano.[5] Hull et al.[6] studied failure mechanisms of a filament wound cylinder subjected to internal pressure. Hyer[7] has evaluated the stress distribution of cross-ply laminated shells under hydrostatic pressure.

This Note presents a theoretical study of the response of filament wound composite shells under internal pressure. Each layer of the material is generally cylindrically anisotropic. By using cylindrically anisotropic elasticity field equations and Lekhnitskii's stress functions, a system of sixth-order ordinary differential equations is obtained. The general expressions for the stresses and displacements in the laminated composite shells under internal pressure are discussed. Two composite systems, graphite/epoxy and glass/epoxy, are selected to demonstrate the influence of degree of material anisotropy and fiber orientations on the axial and induced twisting deformation. Stress distributions of $[45/-45]_s$ symmetric angle-ply fiber-reinforced laminated shells are shown to illustrate the effect of radius-to-thickness ratio.

Analysis

Consider a laminated cylindrical shell consisting of fiber-reinforced laminas subjected to internal pressure. It is assumed that the axis of anisotropy coincides with the longitudinal z axis. The shell is assumed to be long enough so that, in the region away from the ends, Saint Venant's principle holds. Consequently, the stress components are independent of the longitudinal z axis of the shell. As a result of axisymmetric deformation, one can establish a system of coupled governing ordinary differential equations in terms of Lekhnitskii's stress functions $F(r)$ and $\Psi(r)$[8,9] for the individual lamina:

$$L_4'F + L_3'\Psi = 0, \qquad L_3''F + L_2'\Psi = \frac{S_{34}A_3}{S_{33}r} - 2A_4 \tag{1}$$

where L_4', L_3', L_3'', and L_2' are linear ordinary differential operations defined as

$$L_4' = \tilde{S}_{22}\frac{\mathrm{d}^4}{\mathrm{d}r^4} + 2\tilde{S}_{22}\frac{1}{r}\frac{\mathrm{d}^3}{\mathrm{d}r^3} - \tilde{S}_{11}\frac{1}{r^2}\frac{\mathrm{d}^2}{\mathrm{d}r^2} + \tilde{S}_{11}\frac{1}{r^3}\frac{\mathrm{d}}{\mathrm{d}r}$$

$$L_3' = -\tilde{S}_{24}\frac{\mathrm{d}^3}{\mathrm{d}r^3} + (\tilde{S}_{14} - 2\tilde{S}_{24})\frac{1}{r}\frac{\mathrm{d}^2}{\mathrm{d}r^2}$$

$$L_3'' = -\tilde{S}_{24}\frac{\mathrm{d}^3}{\mathrm{d}r^3} - (\tilde{S}_{14} + \tilde{S}_{24})\frac{1}{r}\frac{\mathrm{d}^2}{\mathrm{d}r^2}$$

$$L_2' = \tilde{S}_{44}\frac{\mathrm{d}^2}{\mathrm{d}r^2} + \tilde{S}_{44}\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r} \tag{2}$$

where A_3 and A_4 pertain to uniform axial deformation and relative angle of rotation about the z axis. The reduced compliance constants $\tilde{S}_{ij}$ are defined as

$$\tilde{S}_{ij} = S_{ij} - \frac{S_{i3}S_{j3}}{S_{33}} \qquad (i,j = 1,2,4,5,6)$$

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*Assistant Professor, Department of Mechanical and Aerospace Engineering. Member AIAA.